$$\begin{cases} \dfrac{\partial V}{\partial t}+\dfrac{1}{2}\sigma\left(\dfrac{\partial^2 V}{\partial S^2}\right)^2 S^2\dfrac{\partial^2 V}{\partial S^2}+(r-q)\left(\dfrac{1-\tau}{1-c}\right)S\dfrac{\partial V}{\partial S}-rV=0, \\ \sigma\left(\dfrac{\partial^2 U}{\partial S^2}\right)=\begin{cases}\sigma^{*-}\dfrac{\partial^2 V}{\partial S^2}<0 \\ \sigma^{*+}\dfrac{\partial^2 V}{\partial S^2}>0\end{cases}, \\ U(x)|_{t=T}=e^{-(T-t)}h\big(S(T)\big). \end{cases} \tag{8}$$

3. NUMERICAL ALGORITHM

According to the reference 5, the solutions of the above nonlinear partial differential equations are regular. Unfortunately, its analytical solution does not exist. The finite difference method can be used to solve the nonlinear partial differential equation. We must set up a discrete grid, in this case refering to time and asset prices. Let T be the maturity of the option and $S_{\max}$ be a sufficiently large asset price, which plays the role of $+\infty$. The grid consists of points (S,t) such that $S=0{:}dS{:}S_{\max}$ and $t=0{:}dt{:}T$, and we use the grid on notation representing the option value with stock price at time t. The Finite difference method is applied to approximate the partial derivatives.

$$\begin{aligned} &\frac{\partial V}{\partial x}=\frac{V_{i,j}-V_{i-1,j}}{\Delta x}, \\ &\frac{\partial V}{\partial x}=\frac{V_{i,j}-V_{i,j-1}}{\Delta x}, \\ &\frac{\partial^2 V}{\partial x^2}=\frac{V_{i+1,j}-2V_{i,j}+V_{i-1,j}}{(\Delta x)^2}. \end{aligned} \tag{9}$$

Let the approximate of the partial derivatives (9) substitute the nonlinear partial differential equations. The pricing formula under the worst condition Equation (7) is shown as follows:

$$\begin{cases} \dfrac{\partial V}{\partial t}+\dfrac{1}{2}\sigma\left(\dfrac{\partial^2 V}{\partial S^2}\right)^2 S^2\dfrac{\partial^2 V}{\partial S^2}+(r-q)\left(\dfrac{1-\tau}{1-c}\right)S\dfrac{\partial V}{\partial S}-rV=0, \\ \sigma\left(\dfrac{\partial^2 U}{\partial S^2}\right)=\begin{cases}\sigma^{*+}\dfrac{\partial^2 U}{\partial S^2}<0 \\ \sigma^{*-}\dfrac{\partial^2 U}{\partial S^2}>0\end{cases}, \\ U(x)|_{t=T}=e^{-(T-t)}h\big(S(T)\big). \end{cases}$$

The recursive format is obtained as follows:

$$V_{i,j-1}=a(i)V_{i-1,j}+b(j)V_{i,j}+c(j)V_{i+1,j},$$

Where

$$a(i)=\begin{cases}\dfrac{1}{2}\Delta t(\sigma^{+^2}i^2-(r-q)i), if\ \dfrac{\partial^2 V}{\partial S^2}<0 \\ \dfrac{1}{2}\Delta t(\sigma^{-^2}i^2-(r-q)i), if\ \dfrac{\partial^2 V}{\partial S^2}>0\end{cases},$$

$$b(i)=\begin{cases}1-\Delta t(\sigma^{+^2}i^2+(r-q)i), if\ \dfrac{\partial^2 V}{\partial S^2}<0 \\ 1-\Delta t(\sigma^{-^2}i^2+(r-q)i), if\ \dfrac{\partial^2 V}{\partial S^2}>0\end{cases},$$

$$c(i)=\begin{cases}\dfrac{1}{2}\Delta t(\sigma^{+^2}i^2+(r-q)i), if\ \dfrac{\partial^2 V}{\partial S^2}<0 \\ \dfrac{1}{2}\Delta t(\sigma^{-^2}i^2+(r-q)i), if\ \dfrac{\partial^2 V}{\partial S^2}>0\end{cases}.$$

Similarly, when it comes to the best condition Equation (8), the recursive format could be also obtained as follows:

$$V_{i,j-1}=a(i)V_{i-1,j}+b(j)V_{i,j}+c(j)Vi_{+1,j} \ ,$$

Where

$$a(i)=\begin{cases}\frac{1}{2}\Delta t(\sigma^{+^2} i^2-(r-q)i), if \frac{\partial^2 V}{\partial S^2}>0 \\ \frac{1}{2}\Delta t(\sigma^{-^2} i^2-(r-q)i), if \frac{\partial^2 V}{\partial S^2}<0\end{cases},$$

$$b(i)=\begin{cases}1-\Delta t(\sigma^{+^2} i^2+(r-q)i), if \frac{\partial^2 V}{\partial S^2}>0 \\ 1-\Delta t(\sigma^{-^2} i^2+(r-q)i), if \frac{\partial^2 V}{\partial S^2}<0\end{cases},$$

$$c(i)=\begin{cases}\frac{1}{2}\Delta t(\sigma^{+^2} i^2+(r-q)i), if \frac{\partial^2 V}{\partial S^2}>0 \\ \frac{1}{2}\Delta t(\sigma^{-^2} i^2+(r-q)i), if \frac{\partial^2 V}{\partial S^2}<0\end{cases}.$$

4. EMPIRICAL ANALYSIS

We use the Chinese 50ETF stock index option to make an empirical analysis. All the data are obtained from the wind database. Over the past six months, the trend of the 50ETF stock index price is shown as Figure 1, meanwhile, the implied volatility is shown in Figure 2. By calculating, we got the adjusted volatility interval [0.169,0.273]. Considering the 50ETF stock index option on December 30, 2016, the 50ETF stock index price S_0 is 2.287, the risk free interest rate r is 0.035, the transaction cost rate k is 0.003, the dividend rate q is 0.0215. We compare the option pricing between our models and the classical BS models under the different strike price, and show the result as follows in Figure 3.

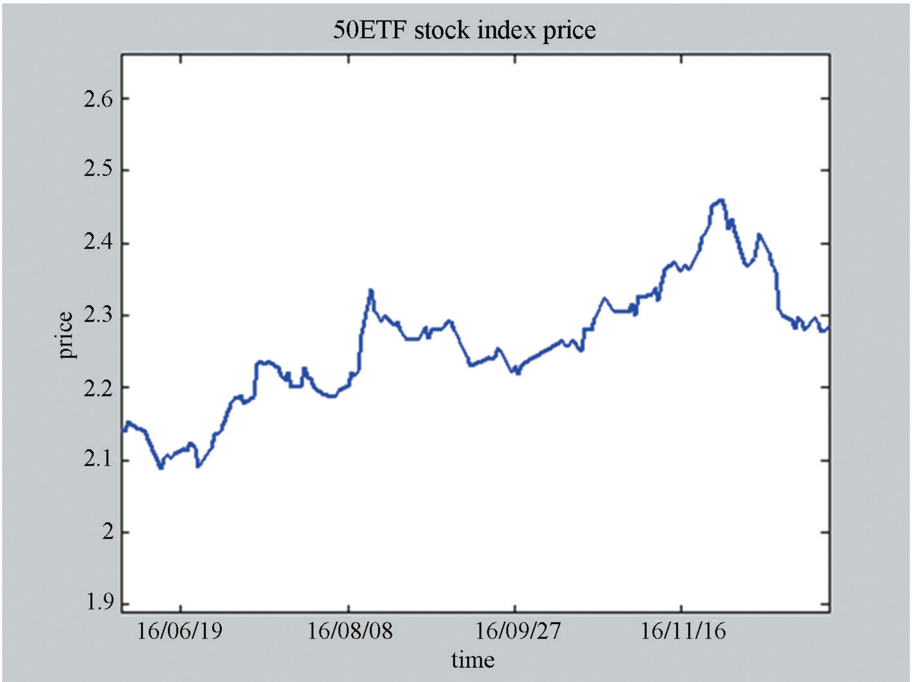

Figure 1
The Price of the 50ETF Stock Index

